Disentangling the Cosmic Web

The Relationship between Walls and Other Cosmic Structures

By: Emma Fuleky

INTRODUCTION

The universe is shaped by the pull of gravity and accelerated expansion due to dark energy. The result is a vast cosmic web composed of:

Clusters: high density regions
Filaments: 1D thread-like structures
Walls: 2D sheets made of galaxies and gas
Voids: empty/low density areas

While filaments and clusters are well studied, walls are harder to detect and therefore less understood. We need methods that are able to identify walls. Because walls and filaments form at different stages of gravitational collapse, characterizing their spatial relationships and comparing their densities can improve our understanding of how:

- dark matter organizes large cosmic structures
- galaxies have evolved
- cosmos expands due to dark energy

Research Objectives: Develop a workflow to identify and characterize walls and their relation to other structures of the universe. Compare walls and other cosmic structures by their density and spatial distribution.

2D VISUALIZATION

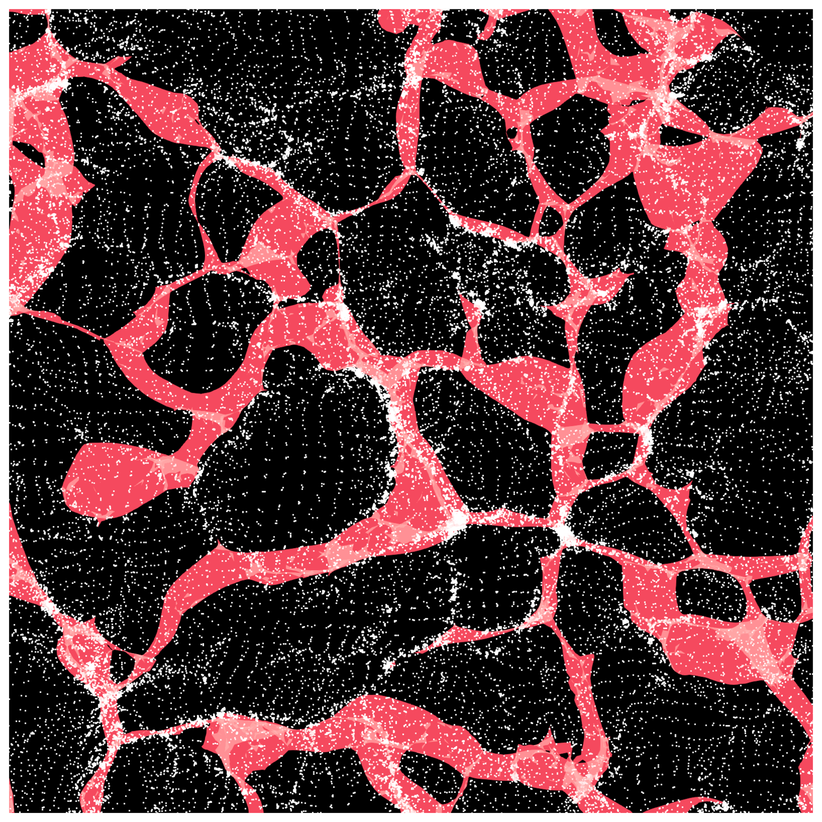

Image created by Emma Fuleky using Paraview 2026 (pvpython)

Figure 1. Walls (red) overlaid on original particle simulation (white) in a 200 x 200 x 10 Mpc^3 (32.6 million light years thick) slab flattened into 2D.

Problem: When analyzing the cosmic web in 2D, walls can be mis-categorized and missed entirely: if orthogonal to the slicing plane, walls look like filaments. If a wall spreads along a slice, its lower density makes it difficult to visualize. **Solution**: As described under methods, I created a DisPerSE workflow to identify each structure, and in 2D, I distinguish them from filaments with solid color.

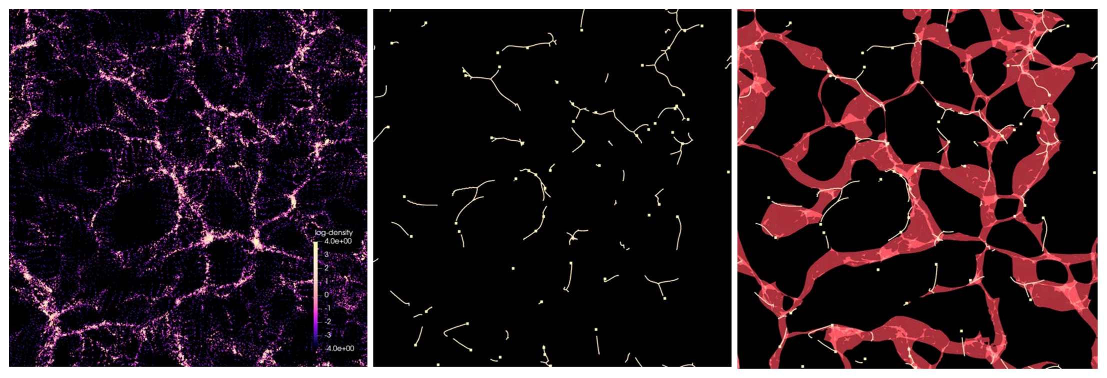

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Figure 2. 200 x 200 x 10 Mpc^3 slab flattened into 2D. Left: simulated particles colored by log-density. Center: filaments (orange) and clusters (yellow). Right: walls (red) with filaments and clusters.

Key Findings: I find that filaments are mostly found inside walls, which is important for our understanding of their evolution. Therefore, the known location of filaments can be used to identify the location of walls.

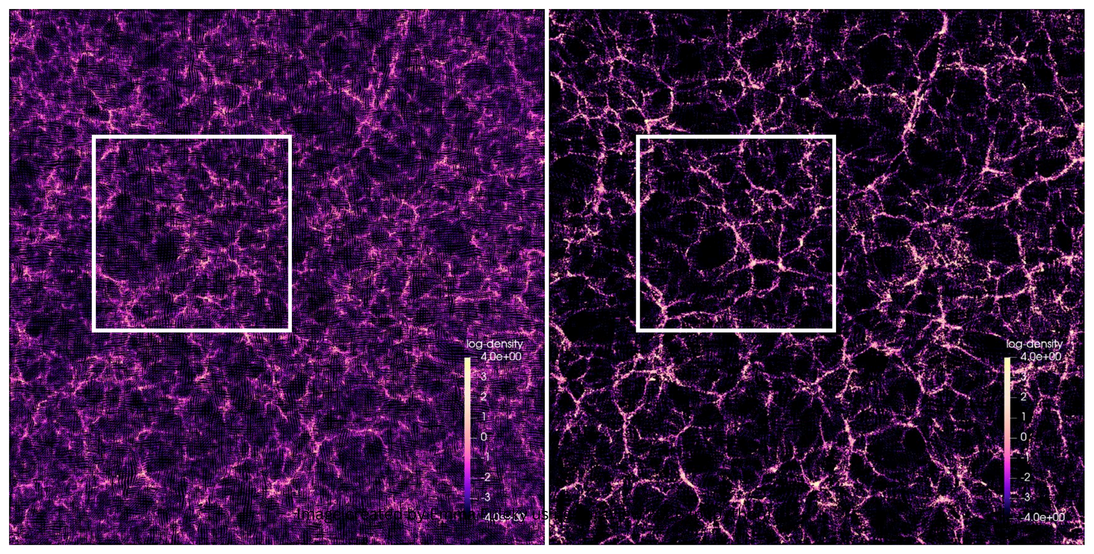

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Figure 3. Density distribution in a 500 x 500 x 10 Mpc^3 slab. Left: ~11.5 Billion years ago. Right: present.

This project compares structures at redshifts z=0 (present day) and z=3 (11.5 billion years ago) and it demonstrates, how the universe went from a seemingly smooth distribution (with little density fluctuations) to highly defined structures because matter primarily drifted from less dense regions such as voids into walls.

DATA & METHODS

I developed the following workflow:
Quijote→ Cropping→ DTFE→ MSE→ Persistence→ Statistics

1.Quijote Simulations: a large collection of N-body simulations describing the evolution and structure of the cosmos. I selected a simulation with a standard set of fiducial parameters at redshifts 0 and 3.

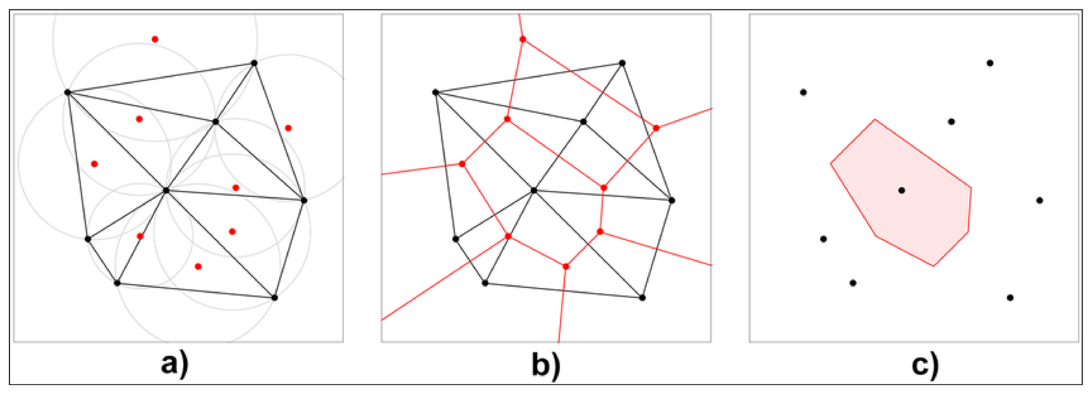
- cube measuring 1 h^{-1} Gpc (3.26 billion light-years) per side with a total of 134 million particles (512^3 particles)
- the simulated particles represent dark matter halos
- data from Quijote obtained via Globus data transfer

2.Cropping:

- due to limited computer memory, the analysis is not able to handle the whole 1 h^{-1} Gpc^3 cube.
- I cropped the whole cube into 4 columns x 10 rows (500 x 500 x 100 Mpc^3 or ~3.5M particles/crop)

DisPerSE: I applied a set of command line utilities for Discrete Persistent Structures Extraction as follows:

3.Delaunay Tessellation Field Estimator: creates a density field from discrete particles. Calculates densities using the inverse of the volume of the Voronoi cell.



Note. From Hafver, Andreas & Agrell, Christian & Vanem, Erik. (2022). Environmental contours as Voronoi cells. Extremes. 25. 1-36. 10.1007/s10687-022-00437-7.

Figure 4. Depictions above show the process of changing a discrete data set into a density field. a) Delaunay Triangulation: no points lie in the circumference of any triangle. The red points are the circles' centers. b) Voronoi vertex: the centers from the triangles become connected by each side's bisector. c) Voronoi cell: set of points that are closer to that vertex than to any of the other vertices. This generalizes to n dimensions. Before tessellation: each point has unit mass of 1. After tessellation: the inverse volume of each cell in which the point is located becomes the density.

4.MSE (Morse-Smale Complex Extraction): identifies critical points in the density field. Critical points are regions where the gradient of the density field is zero

- maxima (highest density): cluster peaks
- saddle points (in 3D there are two types)
  - 1D saddle path to maxima: filaments
  - 2D saddle sheet to maxima: walls
- minima (lowest density): voids

5.Persistence (Simplification): persistence is used to tell important, robust structures apart from small features that are potentially just noise. Features below a certain persistence threshold (3.5 sigmas) are smoothed out.

6.Slicing: every crop is sliced into 10 slabs, which are then used for visual analysis. Thickness of the slab is important:

- too thick = densities will average over and look homogeneous
- too thin = not enough information to easily distinguish structures

7.Statistics: reassemble the analyzed individual crops into a 1 Gpc^3 cube, and determine which cosmic structure each particle belongs to.

- obtain the distribution of densities and log-densities across cosmic structures
- calculate summary statistics for the distributions

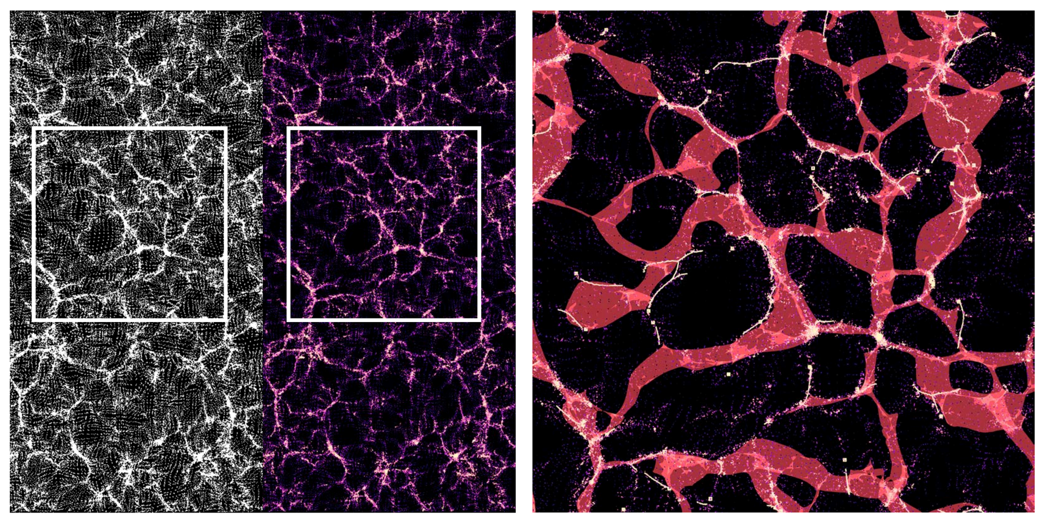


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Figure 6. Left panel: Original particle simulation in a 500 x 500 x 10 Mpc^3 slab (left half) and the same slab colored by log-density (right half). Right panel: Walls (red), filaments (orange), and clusters (yellow) in the same slab. The 200 x 200 x Mpc^2 white squares mark matching areas; the same area is also shown on the right panel.

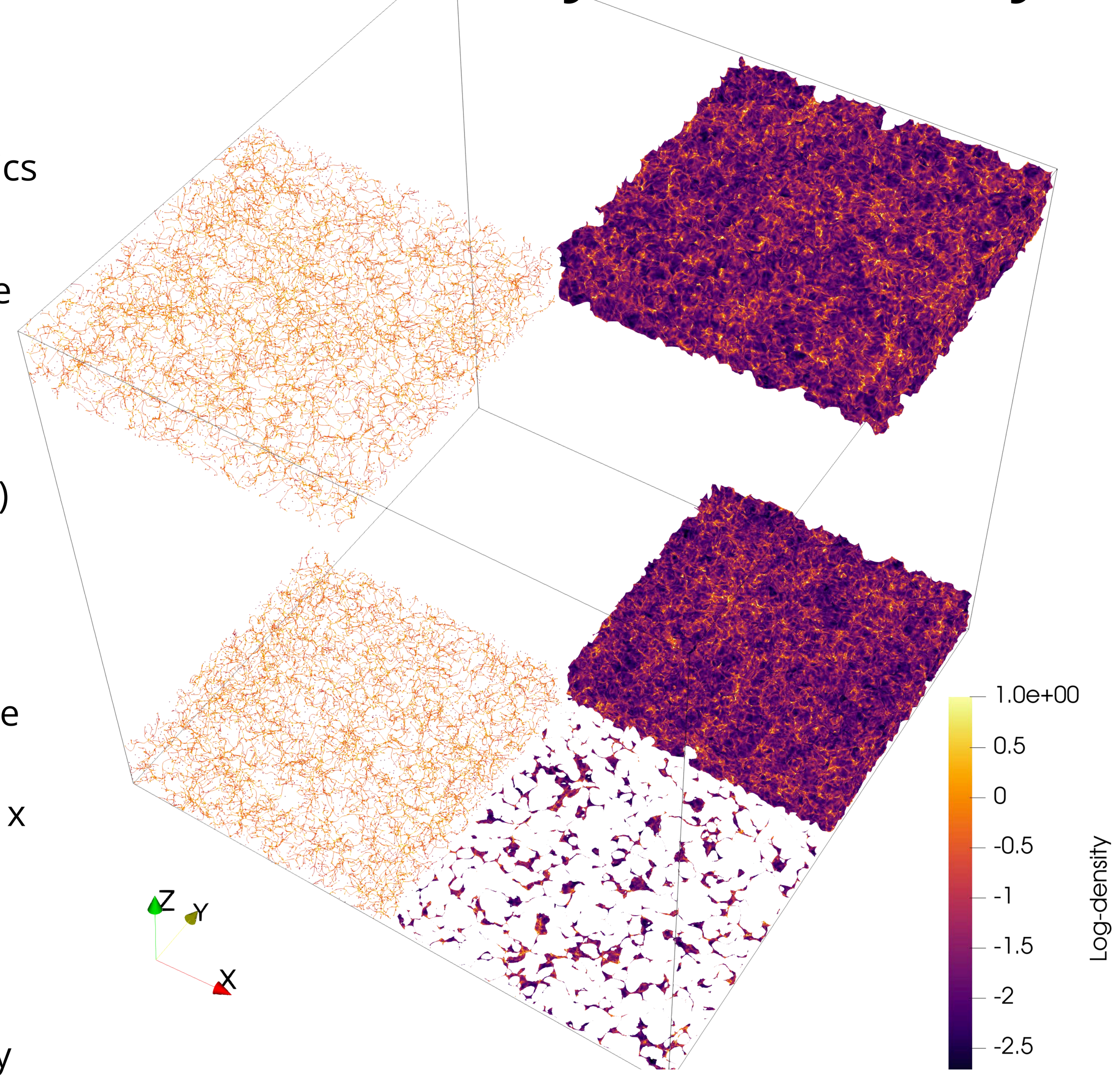

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Figure 5. Outline of the entire 1 Gpc^3 cube and crops sized 500 x 500 x 100 Mpc^3, with filaments on the left and walls on the right. Foreground also shows a 500 x 500 x 5 Mpc^3 slice of walls.

DISCUSSION

Existing Hessian-based classifications (Cautun et al. 2013; Hahn et al. 2007; Libeskind et al. 2018) have shown that filaments are typically embedded in walls and clusters at filament intersections. My project quantifies this relationship using a methodologically distinct topological approach (DisPerSE, Sousbie 2011). Two findings from my workflow independently corroborate the Hessian-based picture: clusters appear at filament endpoints and intersections, and no cluster connects directly to a wall, confirming the expected structural hierarchy.

Key Quantitative Findings:

- ~99% of filaments lie within walls
- filaments and clusters inside walls are denser than those outside in both redshifts, suggesting walls actively concentrate matter
- between z=3 and z=0, walls densified by ~10x, filaments by ~20x, and clusters by ~100x (medians)
- walls, containing ~51% of all particles at z=0, are essential in defining the geometry of the cosmic web

CONCLUSION

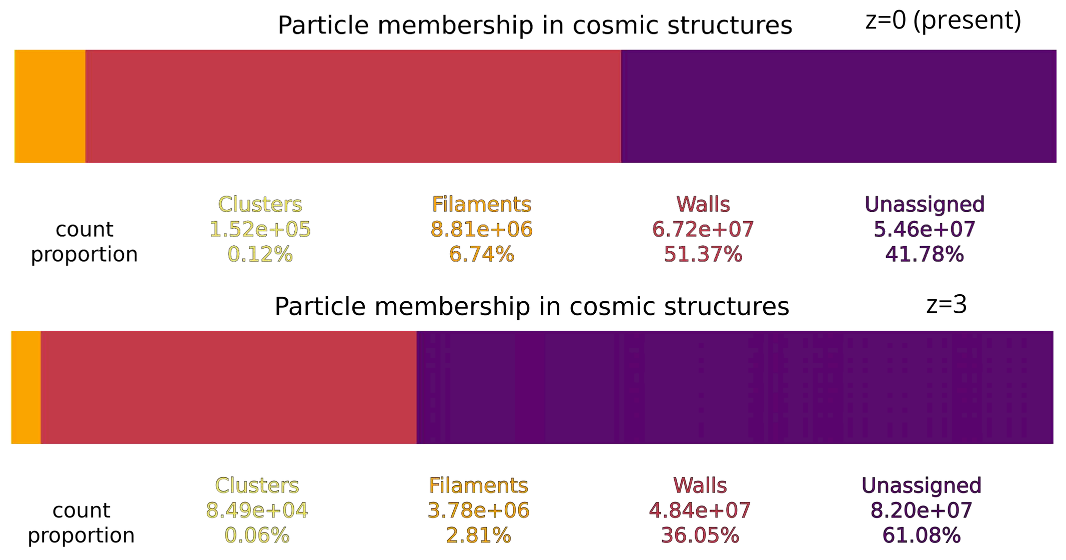
Implications of this Project:

- walls are structurally fundamental
- gravitational collapse funnels matter along the wall → filament → cluster path
- the order-of-magnitude median density difference between walls and filaments suggests that filament galaxies exhibit more mergers and earlier onset of star formation than more isolated wall galaxies

Next Steps:

- analyze multiple Fiducial simulations (with different random seed) to attach uncertainties to results
- analyze simulations from different cosmological models to illustrate the impact of various parameter values and validate models against the real universe
- in this project, clusters are identified as the most extreme density points rather than their full extent: apply a group-finding method (e.g. Friends-of-Friends algorithm) to map the full boundary of each cluster, giving a more complete picture of how much mass and volume clusters truly contain
- cross-validate against Hessian based methods on the same data, using the framework of Libeskind (2018)
- run persistence-threshold sensitivity tests

QUANTITATIVE ANALYSIS



z=0 (present)

	Clusters	Filaments	Walls	Unassigned
count	1.52e+05	8.81e+06	6.72e+07	5.46e+07
proportion	0.12%	6.74%	51.37%	41.78%

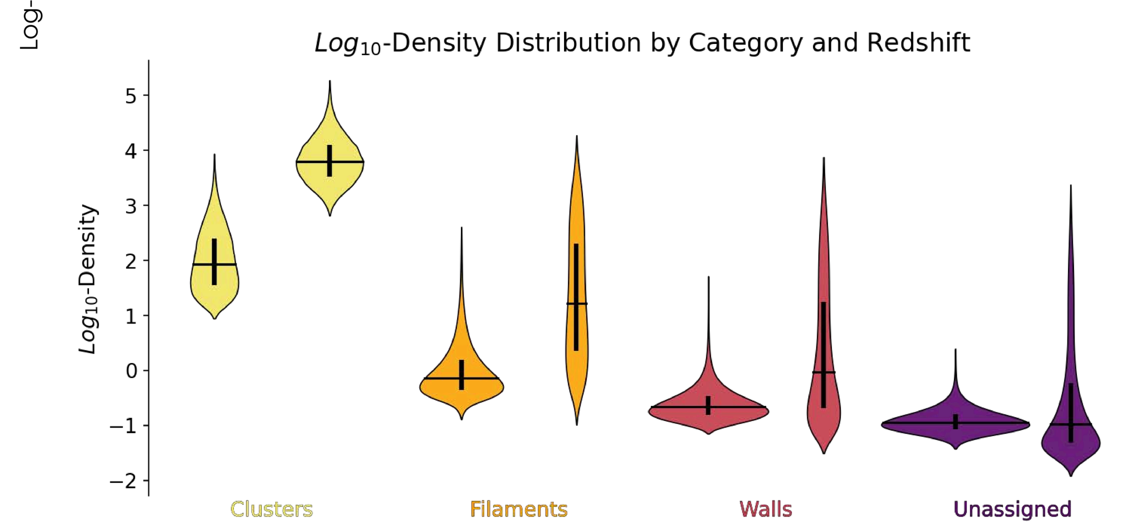
z=3

	Clusters	Filaments	Walls	Unassigned
count	8.49e+04	3.78e+06	4.84e+07	8.20e+07
proportion	0.06%	2.81%	36.05%	61.08%

Graph and table created by Emma Fuleky using python

Figure 7. Proportion of particles by category. The color coded legend lists the count and proportion of particles associated with each structure in the universe. The unassigned category can be considered voids. The proportion of particles in clusters is <0.2% and thus it is not visible in the chart (unless you squint really hard).

Key Findings: Clusters are peak densities, and therefore relatively rare (<0.2% at both z=0 and 3). Walls spread over two dimensions and therefore contain many simulated particles (e.g. up to 51% at z=0) highlighting their importance. Due to gravity, over time more particles merged into clusters, filaments, and walls, while the number of "unassigned" or void particles has declined. Walls play an essential role in this redistribution of matter.



	Clusters z=3	Clusters z=0	Filaments z=3	Filaments z=0	Walls z=3	Walls z=0	Unassigned z=3	Unassigned z=0
max	3.937	5.276	2.609	4.275	1.714	3.878	0.393	3.378
q75	2.398	4.093	0.195	2.301	−0.472	1.246	−0.799	−0.233
mean	2.004	3.822	−0.012	1.347	−0.608	0.333	−0.918	−0.584
median	1.935	3.800	−0.132	1.222	−0.665	−0.026	−0.943	−0.975
q25	1.551	3.524	−0.352	0.359	−0.816	−0.686	−1.067	−1.322
min	0.944	2.814	−0.887	−0.985	−1.150	−1.503	−1.425	−1.917
sd	0.562	0.408	0.496	1.183	0.310	1.241	0.215	1.067
skew	0.122	0.055	0.242	0.105	0.184	0.289	0.117	0.366
count	8.47e+04	1.52e+05	3.77e+06	8.80e+06	4.83e+07	6.70e+07	8.18e+07	5.45e+07

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Figure 8. Each violin-plot shows the distribution of log10-density for a cosmic structure. Below each plot are the corresponding summary statistics.

The density of cosmic structures is inversely related to their dimensionality, with clusters (0D) being more dense than filaments (1D), which are more dense than walls (2D), and "unassigned" or voids (3D). Over time the density of 0D-2D structures increased due to gravitational collapse, and became more heterogeneous with a wider range.

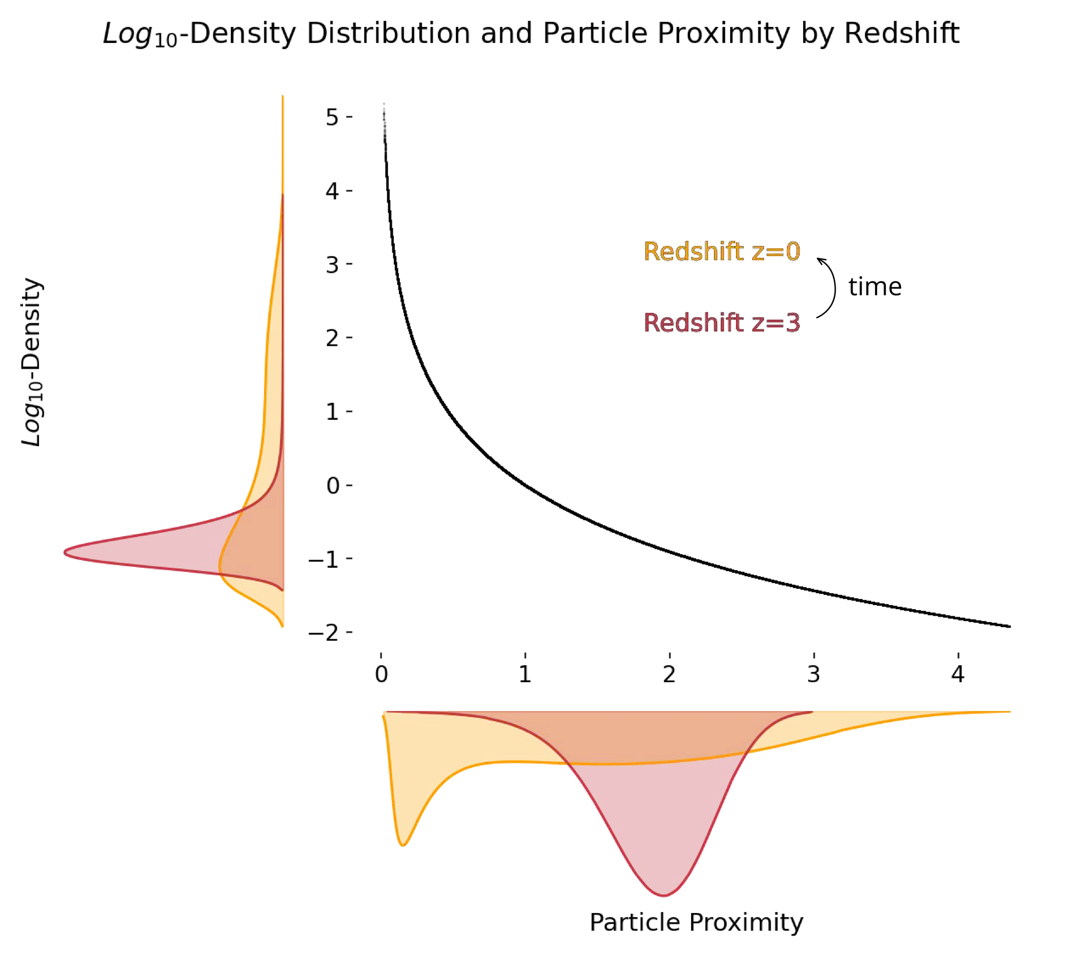

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Figure 9. Nonlinear inverse relationship between particle proximity and density. Greater heterogeneity of distance between particles and skewed density implies clustering of matter. Redshift z=3 is ~11.5B years ago, z=0 is present.

Density is the inverse of the volume of the Voronoi cell. I calculate mean particle proximity (x) as the cube root of the volume of a Voronoi cell. Hence the relationship between density and particle proximity is $\rho = x^{-3}$. Using $x = \rho^{(-1/3)}$ and $y = \log_{10}(\rho)$, Figure 9 illustrates the gravitational collapse during the universe's evolution: particle clustering (bunching near x=0) and increasing density (positive skew along y).